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Learning as fuzzy structure: Integrating quantum nature of the learning processes measurements and teaching experience.

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In this research, a methodological tool that can be used to make learning measurements over a numerical conception completely different is proposed. The new tool uses the concept of built-in measurements, which represent, in a closer way, the teaching and learning process nature. The process is called *built-of,* because the learning quantification involves the teacher's experience and his or her daily relationship with the student. In the integration of the quantum nature of measurements with the teacher's experience, we understand learning as fuzzy numerical structure due to the theoretical assumption about the imprecise nature of measurements from an abstract phenomenon. This conception and methodological proposal enhances modeling projections, analysis and understanding of the learning process.

Key words: measurement process, learning, fuzzy number

INTRODUCTION

The fuzzy set theory provides a mathematical treatment to some vague linguistic terms, such as ``about'', ``around'', ``close'', ``short'', among others. For example, from its point of view, numbers are idealizations of imprecise information expressed by means of numerical values. For example, when the height of an individual is measured, a numeric value is registered including some inaccuracies. Such inaccuracies may have been originated by the measurement instruments, human limitations, biased prior information among many others causes. If the ``real'' value of the height is represented by the number *h*, maybe it would be more correct to say that the value of the height is approximately and not exactly *h* (Barros, 2010). As proposed by Coppi et al. (2006), the fuzzy theory may provide an additional value to the statistical methods, due to the uncertainty inherent to the observable world and its associated information sources are combined beyond the traditional probability theory. For example, Tanaka (1982) introduced the concept of fuzzy regression, while Wunsche and Nather (2002) characterized the least squares method for fuzzy random variables. Moreover, Choi (2006) extended the fuzzy regression model for a censoring scheme. Dubois (2006) discussed some issues about possibility theory and statistical reasoning, and recently, Arabpour (2008) developed some theoretical elements regarding parameter estimation in fuzzy regression models. The connection between the estimation of parameters and the fuzzy theory has been studied by several authors. Cheng (1993) studied fuzzy systems using confidence intervals, and Chiang (2001) analyzed linear programming problems using confidence sets. Geyer (2005) established a relation between the concept of p-value and fuzzy structures, while Pachami (2006) introduced the concept of fuzzy confidence intervals.

To understand the learning process, several mechanisms or methodologies have been proposed, which seek to quantify learning (Author 2012), where it is set out that to measure learning it must be considered as a dynamic system constantly interacting with different realities. If in a concrete way there is already imprecision on measures, it is likely to be even more imprecise with this condition or typical element from abstract nature of man. For this reason, this work presents a new numerical structure, allowing to walk towards integral quantification of learning which is generated with interaction apparently without bonding: the experience that the teacher acquires in the classroom and the recent progress in mathematical-statistics models, specifically the proposal of numerical structures, it is transformed into a great step during the modeling process and understanding of the learning process phenomenon. These structures are known as fuzzy numbers.

LITERATURE REVIEW

In the understanding process of the learning phenomenon, it is possible to distinguish a varied range of proposals. For example, Cabrera et al (2010) proposed a mathematical-statistical methodology, in which the answering time to a stimulus is considered as a significant information element in order to know whether the learning structure is consistent or not. Fernández (1997) proposed the normal distributional model, as a representative, almost as a rule, from the learning process, however, this might be obsolete under the new projections of statistical modeling. Arellano-Valle (2005) reported that the data or the measures are those that have to give their model and also those in which the researcher does not have to force them to assume a determined behavior, leaving aside the symmetry assumptions and infinity supports. Authors (2012), based on Ojeda (2003),proposed to recognize the dynamic and interacting nature of a person and make it part of a model which considers to recognize these features, but on the basis of precise measures depending on the scores from a test. Nevertheless, Crombach (1951) indicated that a test is subject to reliability and validity which, evidently, strongly weakens the basis of measures precision, as an alternative to improve validity and reliability. García (2002) proposes a hermeneutic perspective, but it is just an improvement of those methods.

The perspective of learning as a fuzzy unit, has a lack of bibliographic references, and it is null in the specific educational area. However, this may have interesting applications in the learning tools or supervised learning (Soto 2011). Acampora (2010), who used the fuzzy view in the theory of system decisions, reported another potential use of this. Another interesting research regarding this growing methodology can be found in Barros (2010).

THE RESEARCH PROBLEM and OBJECTIVES

Recognizing the incapability of measuring learning precisely and the continuous seek of integral measurements methodologies, which allow to join co-variables and determining the significance of their effects, the problem of our research is: "to propose a methodology of a quantum representation of learning, which represents in an integral way a measurement based on the fuzzy numerical structures, which characterization is based on the teacher's pedagogical experience".

Objectives

- To describe the fuzzy structures.
- To incorporate the teaching experience on the characterization of a fuzzy number in the learning measurement process.
- To promote the interaction between objective quantum methodologies and subjective methodologies.
- To promote a line of research based on the conception of learning as a fuzzy structure.

RESEARCH METHODOLOGY

The methodology is propositional, aiming to beginning a new line of research about integral educational measurements and understanding the learning phenomenon.

Phases of Research

1. Preliminary analysis.
2. Quantification of learning based on the integration of the teacher's experience and numerical fuzzy structures.
3. Application.
4. A *posteriori* analysis and evaluation.

SOME PRELIMINARY ANALYSIS

The statistical modeling

Statistical models have been used in a wide range of situations. For example, to solve specific problems in engineering and different scientific areas, and constitute the basis of the theoretical formulation of inference and most of the statistical methods (Arellano-Valle, 2005: 93-94). Nowadays, statistical modeling has methodological and technological backups that give a great viability for an educational development in modeling. A statistical model is a platonic conception of theoretical that, in a very generic way, can be seen as a mental constructor that aims to study and better understand a phenomenon in which a cause and effect relationship underlies (Ojeda, 2003: 71-72). Understanding this section is essential to understand the meaning of this work, since one of the main objectives of education is to understand the phenomenon of learning, this phenomenon has an ideal model that perfectly explains this. However, in the process of proposing models, it should be increasingly considered characteristic elements corresponding to data we observe. In this sense, a proposal which can be used in the children quantification learning process, incorporating a suppose which represents nature's learning and its measurement.

Fuzzy Number

A fuzzy number is a numerical structure different from the generally used. This is specifically characterized by the incompletion of two typical characteristics of the Geog Cantor Set Theory, which are the contradiction law and the principle of the excluded third. Which set, if A is a set contained in a universal set U, thus $A \cap A^c \neq \phi$ and $A \cup A^c \neq U$, respectively. The unfulfillment of these two laws escape from our true or false logical system (Hailperin, 1986), because this perspective leads to degrees of veracity or falseness, that is we do not only have two characterization alternatives of a preposition, but also an infinite set of possibilities. To see these structures in depth see Arabpour and Tata (2008),

Formally, a fuzzy set is a collection of ordered pairs, say (x; F(x)) where the first component x represents a real number (x in IR) and the second component F(x) represents a defined function in x, which assumes values in the unitary interval [0,1] ($0 \leq F(x) \leq 1$). This function F(x) is called membership function and it is used to quantify the belonging degree or veracity of the observed *x* value. Note that in the Aristotelian logic there exist only two truth values, that is, a proposal is true or, in an exclusive way, it is false. In that case, the membership function would only generate two values: one or zero, and it is known as a characteristic function. Therefore, a fuzzy set is a generalization of the Cantor Set Theory and of the Aristotelian logic (Bradford 2011).

The fuzzy set theory is based on the logical of multiple values. For example, if the set *B ={1,2,3,4}* is a conventional set, each element has the same belonging degree to the B set, which means that *F (x) = 1*; for all $x \in B$. Now, the difference with a fuzzy set *A* is that not necessarily F (x) = 1, considering $x \in B$. Other examples and technical developments can be found in Barros, 2010.

As a particular situation for fuzzy sets, Hwang (2011) and Dubois (1980) define the concept of normal fuzzy set which they called fuzzy number. The characterization of this particularity proposes that if there exists a unique par of the form *(x , 1)*, that is if only a pair of values which constitutes the fuzzy set, has as a value in membership the real number 1, then that fuzzy set is a fuzzy number.

In our initial proposal, we will assume that the quantum observations of the measurement learning process, are fuzzy numbers, existing an x which satisfies F(x)=1. Which relates the traditional methodology of learning numerical quantification with our purpose.

In a functional way, a fuzzy number is represented by

$$A(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right) : x \leq m \\ R\left(\dfrac{x-m}{\beta}\right) : x \geq m \end{cases}$$

where *m* is called center value of the fuzzy number *A* and, α and β are called left and right propagation, respectively. From now on, we will represent a fuzzy number as $A = (\alpha, m, \beta)_{LR}$ where the subscript LR indicates that we must consider the form of the membership function to the left and the right of *m*. As a particular situation, if $\alpha = \beta$, then the fuzzy number $A = (\alpha, m, \beta)_{LR}$ will be called symmetric fuzzy number (Zimmermann, 1996).

5.3 Teaching experience and membership function

In fuzzy methodology, the researcher or expert experience has a meaningful contribution in the characterization of a membership function. So, under our context, the teachers, helped by their daily experience with the students, is who will value and select the function that best represents the student's learning. In Figure 1, some basic graphic forms of membership are presented, although nowadays it is been working on increasing this alternative number of modeling. In curve 1 of Figure 1, it

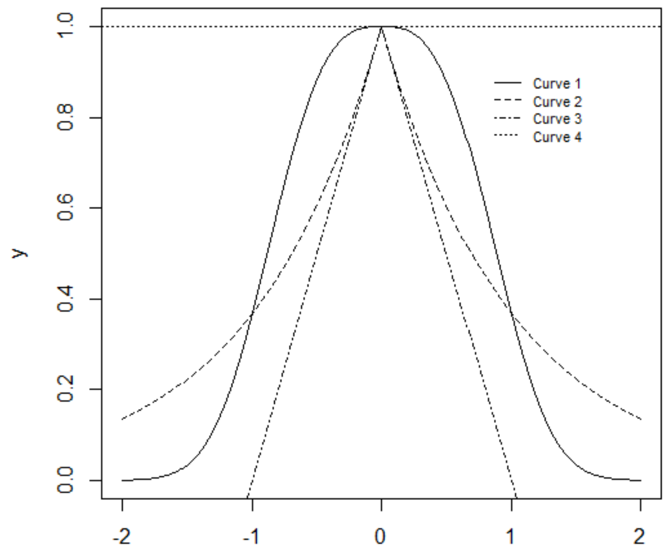
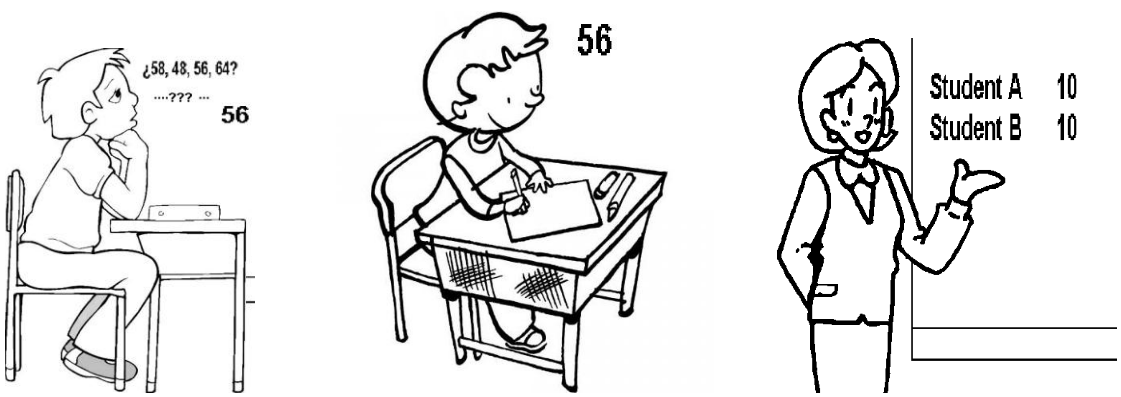


Figure 1. Different forms of the membership function.



Student A Student B Same Quantification

is observed that the decrease of the belonging degrees from neighboring elements to the observed value slowly decrease, which it does not happen in curve 2. Curve 3 it is considered as the simplest situation and it is generally used for its simplicity of calculus, on it the decrease of the belonging degrees is lineal. This membership function is known as triangular and in this particular case is symmetric triangular. Curve 4 has the intention of visualizing a conventional set, whit all the elements having the same veracity or belonging degree.

LEARNING QUANTIFICATION BASED ON INTEGRATION OF THE TEACHER'S EXPERIENCE AND NUMERICAL FUZZY STRUCTURES.

A subset of real numbers is generally used to quantify learning. The traditionally used learning quantification, is a process which tries to objectify the measure and make them comparable. However, it is worth considering, will this numerical label quantify learning in a precise way? The answer is no, because measurement or quantification processes are generally associated to tests whose reliability and validity are questionable. On the other hand, accuracy brings the concept of stability, that is, learning quantification would not vary among tests, however, it varies in a same content. Hence, it is necessary to propose methodologies which help to improve this process, which does not mean changing all this theoretical developments but to incorporate other relevant information in the measurement process. For example, Cabrera et al. (2010) stated that it is not sufficient to quantify whether an answer was correct or not, but there is also a temporary factor which is affecting the answer consistency, which is called by them as answering time for a stimulus. So, summarizing all the teaching and learning process is a quantum symbol, it is a mathematic-statistical methodology which in many times can be malign and overwhelming. All this, does not mean that real numbers are not a good methodology or a bad procedure, but it is still very far from the real representation of the complex system which is learning measurement. That is why we believe that evaluating mechanisms must open to new structures which combine traditional information of a real number and additional information which enrich the measurement.

When a student takes a test, and specifically a question, it is not only the correct answer the one that is in their cognitive structures, but there are many answers competing and it is the student who, as part of the learning and teaching process, must differ and chose one. As an example, In a simple experiment would be done with 100 students, where they are asked to "automatically answer how much is it 7 multiplied by 8", it is interesting to observe that a high percentage answer a different amount of 56, which it does not mean that they do not know but, they need more time to discriminate. That is, there are some other values that belong to these possibilities of answers but are a reflection and induction which inducts them to their answer. That is the way the students generates in their cognitive structures set of possible answers where each element of that set starts acquiring veracity degrees while they make the reasoning process. It is precisely in this process where an answer is selected as real, that is, the answer the students consider as correct.

In this context fuzzy structures have the property of modeling all this dynamic, where if it is contextualized we have, the membership function characterized in section 5.3, models the behavior of the veracity degrees from those possible answers generated by the student at the moment of the test, which will be technically identified as belonging group and all those answers which have as a belonging group 1 or equivalently 100% of veracity for the students, are the ones we observe.

From now on, the additional contribution in this measurement process will be done by the teacher, who is directly involved with this teaching and learning process, and is him the one selecting or proposing an explanatory model of the belonging groups. For example, let's suppose a student A presented difficulties when giving his or her answer, checking it many times, this means that it exited a set of possible answers which belonging degrees were high and they were competing to become the real student's answer. In that case, the membership function presented in Curve 1 from figure 1 would better represent this context. Now, let's suppose a student B, who is confident when selecting an answer, and also does not have doubts on its veracity degree, the membership function which best represents this process can be observed in curves 2 or 3, where it can be shown that the veracity degrees of other possible answers strongly decrease. This quantum integration process turns out to be quite informative and interesting, because the students A and B's situations, do not mention that the answer they gave has been correct but they do mention the structuring of the answer selection model. It is interesting to consider what it means a student on situation B when his or her answer has been incorrect, for it can be the reflection of a solid conceptual structure but mistaken. This means that the student understood the concept and its conceptual logic is consistent to him, but in a wrong way. For student A's case, giving a wrong answer can represent a maximum lack of understanding.

So, to go in depth regarding fuzzy structures, their numerical effectiveness and mathematical formalization, it is recommended to read Zadeh (1978).

APPLICATION

Our application is simple, but it will allow to show the effect and the difference between the learning quantification process with fuzzy numerical structures and the traditionally used method. We will consider 4 students A, B, C and D who took a test and we will specifically analyze their answers from one test question. The set question is: How many divisors number 12 has? The given answers are, respectively: 6, 5, 6 and 6.

From a quantum traditional perspective it can be said that:

- If the test was made up just for that question, students A, C and D would have the same learning quantification and evidently for student B, this will be minor.
- Learning achieved by students A, C and D is better than the one from student B.
- If the class was formed just for those 4 students, we will say it is a group relatively homogeneous.
- Three students succeeded the whole test and one did not.
- Teacher's methodology has a 75% of success.

The previous observations are based on the fact that the test must be a well formulated instrument and with all the desirable metric characteristics.

Under traditional methodology, factors like the emission of the answer process, enough timing for the given answer, consistence of the answer and clarity of the conceptual construct, among others; can be difficultly shown with the information given by the classical numerical quantification.

Note that, a consistent answer does not mean it is correct, but the conceptual construct the student created presents a consistent structure, however, it can be a

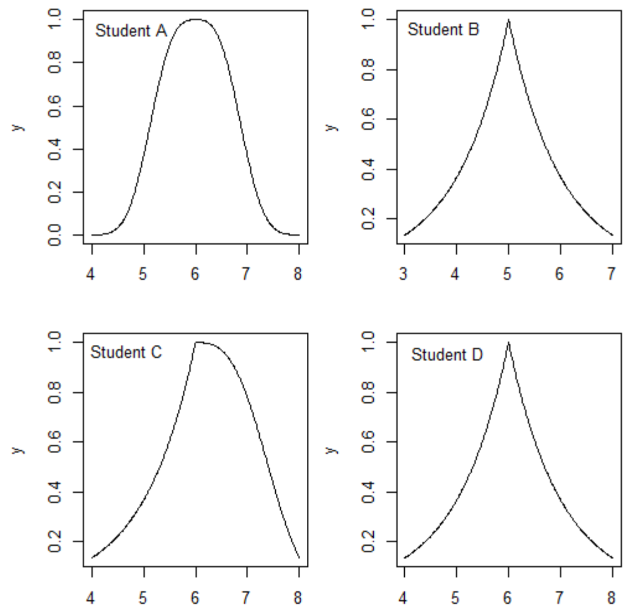


Figure 2. Forms of membership function to the quantifications of students A, B, C and D.

totally closed architecture. For example, when solving the problem, $x^2 - 1 = 0$, the student can have a conceptual structure depending on the procedure $x^2 - 1 = 0 \Leftrightarrow x^2 = 1 \Rightarrow x = \sqrt{1} \Rightarrow x = 1$, which is understood as solid and consistent, however, its consistence it is supported on a wrong construct.

Now, from the fuzzy quantum perspective, the student's answers can be represented as in Figure 2. Under the fuzzy quantum integral analysis approach it can be stated that:

- The traditional quantum method is part of the proposed fuzzy methodology.
- **Student A**, gives a correct answer. Nevertheless, the discrimination process was complex for it is possible to visualize many wrong answers with strong veracity degrees. We could suspect that the time this process required helped with discrimination and selection of the correct answer. It is possible to conclude that this student can present concentration problems.

- **Student B** gives a wrong answer. However, the answer emission process shows strength and consistence, not existing other alternatives with high veracity levels, which allows us to conclude that the student's conceptual constructor is consistent but mistaken. We could say it was a random answer and it could be the reflection of unknowing the concept. An analysis from this perspective allows to naturally make the following assumptions: Maybe it is due to the fact that 1 it is not being considered as a general divisor or perhaps the concept of divisor the student embraced must be minor than the analyzed, among others.
- **Student C**, gives the answer in a correct way, although the emission process is quite interesting since the values which are minor than 6 definitely were not candidates or distractors with significant veracity levels. On the other hand, there are values higher than 6 as possible candidates which will allow us to assume, for example, that the student can have a confusion between the concepts of multiple and divisor, besides of the fact of suspecting that divisors can be numbers higher or equal to the number in question.
- **Student D,** gives a correct answer and shows consistence on the process, in a way that if we dismiss a random answer, this will reflect a clear, coherent and true conceptual construct.

Finally, let's think on the tracking and evolution of a **student B** during a semester in which its learning quantifications, mark 1, 2 and 3, are exactly the same to 8. According to a traditional analysis we would conclude that THERE IS NO EVOLUTION, whereas from the fuzzy integral representation perspective presented on Figure 3, we observe that there is an evolution, that there are changes, that the student's conceptual structures begin to

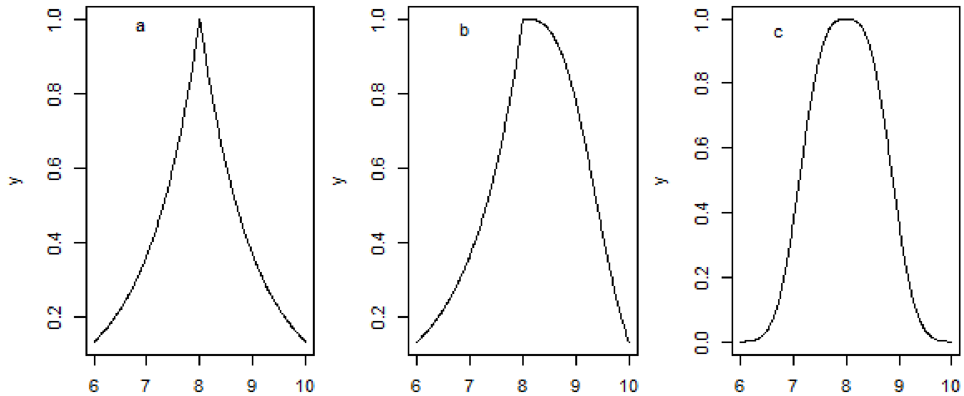


Figure 3. membership functions for notes in the first half of the student E.

weaken, that the student is not reflecting consistence on the answers and the projections according to this logic are unfavorable. Therefore, as teachers, we have the duty of reaction.

Finally, what is interesting from this proposal is its connectivity with the conventional analysis and the knowledge the teacher has over the student's learning, as it is presented on the application.

CONCLUSIONS

From the probabilities point of view, the fact of the evaluation actually made about learning coincides with what the student really known or learned is almost void, that is the probability that the number which we label learning equals real learning, is zero. So, assuming that learning is a fuzzy structure is a much more concrete proposal which allows conventional facts interact with experience and the teacher's daily contact, hence, the impact this methodological proposal has is undeniable.

This proposal promotes a change of mind, in which the conception of learning nature changes and it is located in a specific context to the nature of structures in which the imprecision of measurements is a fact, considering the relativity and uncertainty of a person.

The teaching experience, in this proposal, acquires real importance, for it is the teacher the one who evidences and experiments on a daily basis the relativism, changes and personal factors which involving a evaluative process.

All of those who are teachers and who are worried about the learning phenomenon generate a hierarchical structure of learning in the classroom, which few times differs from the quantifications we observe in a test and it is just that phenomenon the one we call experience, which we integrate in this proposal. We bear in mind that membership functions or possibility models we have to offer the teach are still limited, but we are still working on that lines.

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