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Investigating Estimation: Influences of Time and Confidence of Urban Middle School Students

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This study sought to determine the estimation processes used by 10 urban middle school students for solving computational estimation problems, and if there was a difference in the estimation processes utilized for straight computation and application problems. An adapted model of the Accessing Computational Estimation Test (ACE) was used to determine the estimation strategies employed by the subjects within timed and un-timed settings. Qualitative methods were used to analyze the data. The timed ACE tests were administered using an interview format and included computation and application problems. The findings indicated that there were no differences in estimation processes for straight computation and application problems; however, students performed better on timed tests for application problems.

Keywords: estimation, computational, application, urban students, middle school

INTRODUCTION

There continues to be an on-going debate among mathematics educators in regards to what mathematical skills are necessary to participate successfully in society as well as finding meaningful employment (Lacey and Wright, 2009). Because many everyday uses of mathematics involve computational estimation, teaching children to estimate meets an important practical need. Estimating the cost of purchases, estimating distances, estimating the tip for a waiter or checking to see if an answer on a calculator is reasonable are tasks that require computational estimation strategies. While computational estimation fulfills practical needs, past research on computational estimation had received little attention in the mathematics education literature (Hanson and Hogan, 2000). However, because of the *No Child Left Behind* (U.S. Department of Education, 2001) legislation, The National Council of Teachers of Mathematics' *Principles and Standards* ([NCTM], 2000), the *Common Core Standards* (National Governors Association [NGA], 2010), and the focus on Science, Technology, Engineering, and Mathematics (STEM) education and careers, an increased emphasis has been placed on the knowledge of numeracy, including estimation (Booth and Siegler, 2006).

Number sense, as defined by the NCTM (2000), is an instructive understanding of numbers. This includes connections between numbers, relationship of numbers, and the enormity of possibilities among numbers. Students should be able to develop a conceptual understanding of numbers, as to move beyond the memorization of traditional algorithms (NCTM, 2007). This ability to conceptualize numbers is required of students to meet success with estimation processes and strategies, since estimation is considered a higher level thinking skill (Van de Walle, 2006). Further, for students to be proficient with computation estimation skills, they must develop an awareness of the same flexibility with numbers required of number sense and computation (Van de Walle and Folk, 2005).

There are a number of estimation strategies that rely on teachable algorithms and invented strategies. However, estimating has an additional intricacy. It requires a deep understanding of the value of numbers, mental thinking and computation, mathematical operations and contextual verification (Cochran and Hartmann Dugger, 2013; Van de Walle and Folk, 2005). Students that are able to estimate successfully show an understanding of the value of numbers and of the operations used. Further, they also are able to judge the reasonableness of their answers (NCTM, 2000).

Because of the mathematical requirements needed to gain meaningful employment and to pursue STEM careers, developing an understanding of students' computational estimation strategies and variables that may play a role in its application, is crucial for improving the teaching and learning of computational estimation (Cochran & Hartmann, 2013). This investigation focused on estimation strategies urban middle school students used, and their ability to produce viable estimates that fall within an acceptable interval. The research questions that guided this investigation include:

1. What estimation processes did eighth-grade students use for straight computation and application problems?
2. How did their estimation processes compare for straight computation and application problems?

LITERATURE REVIEW

Computational estimation requires making reasonable guesses as to the approximate answer to arithmetic problems without or before actually doing the calculation (Dowker, 1992; Van de Walle and Folk, 2005). Reys, Rybolt, Bestgen and Wyatt (1980) defined computational estimation as the interaction of mental computation, number concepts, arithmetic skills including rounding, place value and mental compensation that rapidly and consistently result in answers that are reasonably close to a correctly computed result. Current literature still references this definition (NCTM, 2000; Van de Walle Karp, and Bay-Williams, 2012). Researchers agree that mental computation and computational estimation is to be accomplished without the use of paper and pencil, or other tools (Dowker, 1992; NCTM, 2000; Reys, Reys, Hope, 1993; Van de Walle, Karp, and Bay-Williams, 2010).

The ability to estimate or make reasonable guesses for computational problems without doing the actual calculations is considered an essential skill among mathematicians and mathematics educator's alike (Cochran and Hartmann, 2013; NCTM, 2000; Rubenstein, 1985). Estimation is more commonly used than exact computation, and is often considered more important than precise calculations (Levine, 1982). Dowker (1992), Van de Walle and Folk (2005), and Cochran and Hartmann (2013) contend that estimation strategies used by individuals will provide an insight into their understanding of mathematical concepts, relationships, and number sense. Lefevre, Greenham, and Waheed (1993) reported that an understanding of place value and how the number system works, and the ability to work with powers of 10 and size comparisons are required for estimation competence. Rubenstein (1985) concluded that a thorough and flexible understanding of place value, basic facts, number operation properties, and number comparisons is needed to develop estimation abilities and skills. Furthermore, they stated that good estimators demonstrated a high tolerance for errors.

Markovitis and Sowder (1994) describe two types of number sense experts: routine and adaptive. Routine experts are able to solve familiar problems quickly and accurately but are not able to invent new procedures because they are not flexible with number use and place value. Adaptive experts can discover rules, invent algorithms and develop flexible uses of numbers. Children who invent and develop informal strategies appear to have a strong understanding of place value, use numbers flexible, and apply informal strategies based on the flexibility of their understandings (Heege, 1985). Bestgen, Reys, Rybolt and Wyatt (1980) acknowledged that estimation was significantly correlated with problem solving, mathematical ability, and verbal ability, and that the ability to compute rapidly was related to the ability to estimate numerical computations. There is a direct relation between how children process numbers and their cognitive ability (Edwards, 1984).

Skilled estimators often create unconventional, but efficient strategies to calculate different quantities (Hope and Sherrill, 1987). Hope and Sherrill (1987) investigated the processes and procedures used by a highly skilled estimator, and found that the student used strategies that were unconventional and largely self-taught. When studying the computational estimation strategies of professional mathematicians, Dowker (1992) concluded several notable characteristics of their performance, including ". . . a high level of accuracy with occasional errors, a tendency to use strategies involving knowledge of mathematical properties and relationships, and great diversity and flexibility as regards the strategies used" (p. 51). Hanson and Hogan (2000) concluded that college students correctly estimated answers to most problems on addition and subtraction of whole numbers, but performed poorly on multiplication and division of decimals and subtraction of fractions. In addition, these students were more successful solving computational problems when compared to estimating answers. Dowker, Flood, Griffiths, Harris, and Hook (1996) found a positive relationship between estimation performance and the number of strategies utilized in actually making the estimations. Sowder (1989) found that good estimators tend to have a high degree of mathematics self-efficacy, and attribute their estimation success to ability rather than effort.

Sowder and Wheeler (1989) proposed four types of conceptual knowledge specific to computational estimation: a) conceptual components, b) skills component, c) related concepts and skills, and d) affective components. Table 1 shows Sowder and Wheeler's (1989) analysis of the four components involved with computational estimation. The conceptual components involve understanding of the role of approximate numbers in estimation, having an acceptance that estimation could involve multiple processes and have multiple answers, and recognizing that the appropriateness of process is dependent on context and desired accuracy. The skills component involves knowing the processes of reformulation (changing the numbers used to compute), compensation (making adjustments during or after computing), and translation (changing the structure of the problem). The skills component requires determining the magnitude of the estimate and understanding the range of acceptable estimates. The related skills and concepts component involves knowledge of place value, and basic facts, properties. Additionally, the related skills and concepts requires the ability to work with powers of ten, compare numbers by size, compute mentally, and recognize that modifying numbers changes the outcome. The affective component involves understanding the usefulness of estimation and having confidence the ability to do and use estimation.

Cochran and Hartmann's (2013) investigation focused on the estimation strategies used by middle school students. They found that students demonstrated a strong reliance on the rounding strategy or opted to use the exact method. Further, the researchers stated that although rounding is a conventional strategy, students must gain an awareness of the flexibility in regards to different estimation strategies.

The determination of the strategies that an individual uses when giving an estimate to a problem requires something other than a written test (Reys, Rybolt, Bestgen and Wyatt, 1982). Many researchers use interviews to ascertain strategies used when estimating (Dowker, 1992; Forrester and Pike, 1998; Hanson and Hogan, 2000). When oral responses are used, students are instructed to relate their cognitive processes verbally. Reys, Rybolt, Bestgens and Wyatt (1982) researched the processes of good estimators and found that interviews give greater insights into the processes used by good estimators. Also, they found that the interviewer can develop a set of probes to encourage the students to further reveal the strategies being used. Research on computational estimation has primarily focused on strategies people use to estimate computation problems.

METHODOLOGY

Participants and Site

This study was situated in an urban middle school located in the southeastern US. The sample emerged from a population of 56 eighth grade students and only those students who returned a signed parental consent form were eligible to participate and be considered for the actual sample of students included in this study. The sample was one of convenience and was comprised of 10 students. The participants for this study were comprised of five African-American males, four African-American females, and one White female student. These students were enrolled in several different courses, including Pre-Algebra (4 students), Algebra 1 (5 students), and Geometry (1 student).

Procedures

Sampling strategy.

Of the 56 students given parental consent forms, after

Table 1. An Analysis of Computational Estimation Components Involved (adapted from Sowder and Wheeler, 1989)

Components	Descriptions
I. Conceptual Components	1. Role of Approximate Numbers 1.1. Recognition that approximate numbers are used to compute 1.2. Recognition that an estimate is an approximation 2. Multiple Processes/Multiple Outcomes 2.1. Acceptance of more than one process for obtaining an estimate 2.2. Acceptance of more than one value as an estimate 3. Appropriateness 3.1. Recognition that appropriateness of process depends on context 3.2. Recognition that appropriateness of estimate depends on desired accuracy
II. Skill Components	1. Processes 1.1. Reformulation: Changing the numbers used to compute I.1.1.1. Rounding I.1.1.2. Truncation I.1.1.3. Averaging I.1.1.4. Changing the form of a number 1.2. Compensation: Making adjustments during or after computing 1.3. Translation: Changing the structure of the problem 2. Outcomes 2.1. Determination of correct order of magnitude of the estimate 2.2. Determination of the range of acceptable estimates
III. Related Concepts and Skills	1. Ability to work with powers of ten 2. Knowledge of place value of numbers 3. Ability to compare numbers by size 4. Ability to compute mentally 5. Knowledge of basic facts 6. Knowledge of properties of operations and their appropriate use 7. Recognition that modifying numbers can change outcome of computation
IV. Affective Components	1. Confidence in ability to do mathematics 2. Confidence in ability to estimate 3. Tolerance for error 4. Recognition of estimation

two weeks, 31 students returned their forms with signatures. These 31 students participated in the timed estimation computational test that is described next. Students who produced reasonable estimates (i.e., estimates that fall within a pre-determined range, acceptable interval) on at least 20 of the items presented on the timed test were given a second parental consent letter inviting them to participate in a computational estimation interview (un-timed). There were 13 students who received invitations to participate and 10 returned their forms with signatures.

Instrument

The Accessing Computational Estimation (ACE) test (Reys, Rybolt, Bestgen, and Wyatt, 1980) was adapted and used for this investigation. The adapted ACE version has two parts: a) a timed set of computational estimation items were presented to the students using Microsoft's PowerPoint software; and b) an un-timed set of computational estimation items were presented to the students in a face-to-face interview setting. These two parts enabled us to explore the influence of time on

Table 2. Summary of Student Performance for the ACE Tests, Timed and Un-timed

Estimation Problems	Acceptable Estimate Intervals	% Students with Reasonable Estimates, Timed Test	% Students with Reasonable Estimates, Un-Timed Test
1. 87,419 92,765 90,045 81,974 + 98,102	430,000 - 460,000	20	50
2. 31 × 68 × 296	600,000 - 634,000	30	80
3. $8127\overline{)474257}$	50 - 62	10	30
4. $\frac{(347x6)}{43}$	42 - 60	10	30
5. $1\frac{7}{8}$ x 1.19 x 4	8 - 10	20	40
6. 30% of 106,409	30,000 - 36,000	20	40
7. 8483 hot dogs @ $.60	4,200 - 5,400	30	50
8. $21,319,908 share equally among 26 teams	700,000 - 950,000	20	40
9. $28.75 dinner bill 15% tip	$3 - $5	30	30
10. More six 32 oz. bottles or eight 10 oz. bottles	Six 32 oz. bottles (180-240 vs 80-160)	50	90

students' estimation processes. Other adaptations to the ACE tests were made to accommodate the students and decrease the potential for distractions, such as reducing the number of items and updating pictures and prices used in the items.

Data collection

The timed ACE test was made up of 35 computational estimation items – 20 straight computation items (i.e., numbers with operations) and 15 application items (i.e., contextualized computation). The straight computation items were presented first, followed by the application items. Students were allotted 15-seconds per computational estimation item to produce and record an estimate. There was a two-minute rest period between the straight computation items and application items.

An acceptable interval (i.e., the range used to determine reasonable estimates) for each computational estimation problem was pre-determined based on potential strategies students might use. The number of reasonable estimates made by students was collected for the timed ACE test. The un-timed ACE tests were administered using an interview format that consisted of three segments: a) producing computational estimates; b) comparing estimates and calculator computations; and c) investigating students' beliefs - attitudinal and conceptual. This report focuses on findings for only the first interview segment. The un-timed ACE test was comprised of 10 computational estimation problems, a subset of the 35 items used in the timed ACE test – five straight computation and five application (consumer-based contexts) items. The rationale for using the closed set of computational estimation problems was to enable one to determine whether students' estimates differed under different conditions – timed (ACE test using PowerPoint) versus un-timed (ACE test via interview).

The un-timed interview process followed a pattern of presenting a computational estimation problem followed by questions until all 10 problems were presented. During the interview, the straight computation items were presented first, followed by the application items and students were asked to think aloud as they produced their estimates. The interviews were recorded and written work saved to capture students' thinking and estimation approaches. After each problem was presented, students were asked several questions, such as: "How confident are you that you've made a good estimate?" and "Do you think the actual answer is above or below your actual estimate? Why?" The computational estimation problems, five straight computation and five application items, are shown in Table 2.

Data Analysis

There are many estimation strategies, for this study we focus on three processes identified by Reys, Rybolt, Bestgen and Wyatt, (1982) – translation, reformulation, and compensation. These processes were selected for use because they are well defined and could be readily observed given the design of our study and the types of data collected. In addition to the three processes we also anticipated students' using rounding and truncation strategies, because these were common strategies taught and observed in the school setting. We used qualitative methods, constant comparisons (Patton, 1990), to search for evidence of students using one or more of the three estimation processes, rounding, or truncation as estimation strategies. Descriptions of these strategies follow.

Translation is defined as changing the equation or mathematical structure of a problem to a more mentally manageable form (Reys, Rybolt, Bestgen, and Wyatt, 1982). An example from the data of a student using translation for problem 1 (see Table 2): "All of the numbers are about 90,000; so all I have to do is multiply it by 5. The answer is 450,000."

Reformulation is defined as changing the numerical data into a more mentally manageable form. There are two types of reformulation: a) front-end use of numbers; and b) substitution of numbers (Reys, Rybolt, Bestgen, & Wyatt, 1982). An example from the data of a student using front-end reformulation for problem 1 (see Table 2): "I would round the first number to 90,000; the second number to 90,000; the third number to 90,000; the fourth number to 80,000; and the fifth number to 100,000 and add all the rounded numbers and get 440,000." A second example shows a student using substitution of numbers for problem 9 (see Table 2): "I would round $28.75 to $30.00 and change 15% to 10% because 10% is easier to do, my answer would be around about $3.00."

Compensation is defined as adjustments made to reflect numerical variation that came about as a result of translation and/or reformulation, and there are two ways to compensate – intermediate or final compensation (Reys, Rybolt, Bestgen, and Wyatt, 1982). Intermediate compensation occurs when adjustments are made during the middle stages of mental computation. Final compensation occurs when adjustments are made at the end of mental computation, a reflective approach after the fact. An example of both types of compensation are evidenced by this student's think aloud for problem 4 (see Table 2): "I would could change 347 to 350 and multiply that by 6 and get about 2000 then divide that by 40 and my answer would be 50 but I think it should be close to 55 because I rounded so I'll just say it is 54."

Rounding and truncation strategies were used in conjunction with the three key processes – notice the use of each within the previous examples used for describing the estimation processes. Rounding and truncation are described because researchers anticipated heavy usage of both strategies.

Rounding (n.d.) is defined as "a process of replacing a number by another number of approximately the same value. . . ". There were two rounding strategies: a) rounding same number of digits (SND); and (b) rounding extracted (EXT) (Reys, Rybolt, Bestgen, and Wyatt, 1982). Rounding SND is traditional rounding, for example 4,792 can be rounded to the nearest thousand as 5,000 or 4, 297 to 4,000. Rounding EXT would extend that, for example rounding 4,792 to 5,000, and then to 5.

Truncation (n.d.) is defined as numbers that are shortened "by dropping a digit or digits." There were two truncation strategies: a) truncation same number of digits (SND); and (b) truncation extracted (EXT) (Reys, Rybolt, Bestgen, and Wyatt, 1982). Truncation SND keeps the front-end digit(s) and replace the number of truncated digits with zeroes, for example truncating 4,702 or 4,207 both can be truncated to 4,000 and 4,573 can be truncated to 4,500 or 4,570. Truncation EXT uses extracted front-end digits, for example 4,506 can be truncated to 4, 45, or 450.

Both strategies are especially helpful for mental computation when coupled with one or more of the aforementioned estimation strategies.

RESULTS AND FINDINGS

This section discusses and summarizes the broader results gleaned from observations made about students' performance on the computational estimation problems for the ACE tests, timed and un-timed. We focus a set of 10 problems that were shared for both test versions and then we consider students' performance on the straight computation versus the application problems for the timed and un-timed tests. Finally, we offer more specific findings that address the research questions using only the un-timed data. These data included students' thinking aloud as they performed estimation for each problem, which were more detailed and could be categorized into the three estimation processes – translation, reformulation, and compensation.

A general observation apparent from the data is that the eighth-grade student estimators appeared to be influenced by the interaction of several factors, including their background experiences, the mathematical operations, and the size of the numbers. The greatest percentage of reasonable estimates (i.e., those estimates falling within the acceptable interval) were for problems 1, 2, and 10 (see last rows of Table 3 and Table 4). For these problems, 60% to 90% of the students (n=10) were able to create reasonable estimates for these problems

Table 3. Summary of the Number of Students (n=10) with Reasonable Estimates on ACE Tests – Straight Computation Problems

	Number of Reasonable Estimates for Straight Computation Items by Problem					
ACE Test Type	1	2	3	4	5	Total
Timed	2	3	1	1	1	8
Un-timed	5	8	3	3	4	23
Both Tests*	6	8	3	3	4	24

* The number of students who created reasonable estimates for timed, un-timed, or both tests.

Table 4. Summary of the Number of Students (n=10) with Reasonable Estimates on ACE Tests – Application Problems

	Number of Reasonable Estimates for Application Items by Problem					
ACE Test Type	6	7	8	9	10	Total
Timed	2	3	2	3	5	15
Un-timed	4	5	4	3	9	25
Both Tests*	5	5	5	3	9	27

* The number of students who created reasonable estimates for timed, un-timed, or both tests.

across both versions of the ACE tests, timed and un-timed. Addition and multiplication were the only operations needed to find reasonable estimates for these problems; two were straight computation problems (1 and 2) and the third was an application problem (10). Even though, problem 1 involved larger numbers (>80,000), the problem was set up using the traditional addition algorithm, which left limited or no ambiguity for what needed to be done. Most grade eight students would hold prior experiences and understanding at least one way to approach this problem for finding an estimate. Problem 2 included smaller numbers, the largest number in this problem was 296 (very close to 300), all of the numbers were very close to a multiple of 10, and the only operation was multiplication. Most eighth-grade students have proficiency with multiplying multiples of 10 using reformulation with rounding or truncation and managing zeroes. The one application problem among these three problems asked students to estimate numbers of bottles of different sizes and which set was larger. Most students likely had life experiences that allowed them to envision what was being asked and familiarity with the unit of measure (ounces) used for bottled beverages, which perhaps contributed to why 90% of the students accurately providing reasonable estimates for this problem.

As anticipated, the students' performance (i.e., able to find reasonable estimates) was better in the un-timed tests than in the timed tests for 9 of the 10 problems (see Table 2). Their performance was the same (30%) for problem 9, an application problem about estimating a 15% tip, given a dollar amount for the cost of dinner. This problem required students to navigate rational numbers (i.e., decimal and percent fractions) using multiplication; a common challenge for students as most middle school mathematics teachers would likely attest.

We now turn our attention from the broader observed results to the specific findings as we address the research questions and present supporting examples from the data. Our focus will be on the estimation processes, but take note in the examples how rounding and/or truncation are ever present is students' thinking aloud about estimation.

What estimation processes did eighth-grade students use for straight computation and application problems?

By far the most used estimation process used to generate reasonable estimates during the ACE un-timed test was reformulation (33), and this holds true independent of the problem type – straight computation (1-5) and application (6-10) problems (see Table 5). Recall that reformulation is the estimation process of selecting numbers to simplify mental manipulation. Consider problem 10 first, because it offers the most straight forward example of students using reformulation for computational estimation. Nine students produced reasonable estimates for this application problem that asked, "Which carton has more soda? Six 32-oz bottles of Coke or eight 16-oz bottles of Pepsi?" All students used reformulation processes, which varied by students. Several approaches used included: "6 x 30 and 8 x 20";

Table 5. Summary of the Number of Students (n=10) and the Estimation Processes used for Determining Reasonable Estimates on ACE Tests

Problem	Number of reasonable estimates (un-timed)	Strategies Used for Reasonable Estimates		
		Translation	Reformulation	Compensation
1	5	1	3	1
2	8	2	6	0
3	3	0	2	1
4	3	0	3	0
5	4	2	2	0
6	4	2	1	1
7	5	3	2	0
8	4	0	3	1
9	3	0	2	1
10	9	0	9	0
Total	48	10	33	5

"6 x 30 and 8 x 15"; or "6 x 40 and 8 x 20." In each of these examples, clearly the first multiplication represents the larger amount of beverage, or six 32-oz bottles of Coke. This example suggests rounding, but in most instances, an argument that rounding was used would not hold. A better argument for some number choices, such as the 40 and 20 pair is that proportionality is maintained while simplifying the multiplication (i.e., a 40-oz is double the size of a 20oz).

For a second example of reformulation with rounding, consider problem 4 ($\frac{(347x6)}{43}$), only three students offered reasonable estimates. Students producing their estimate by rounding 347 to 300, adjusting six to five, and either rounding 43 to 40 or adjusting 43 to 50. However, two students did $\frac{(300x5)}{50}$ to get an estimate of 50 but compensated to 55. While 55 falls within the acceptable interval for problem 4, the students' computations do not support their estimate and it is unclear from their explanation how they arrived at 55. Seven students were unable to generate reasonable estimates for this problem due to computation errors. Clearly, computation challenges interfered with students' estimation processes.

The translation (10) estimation process was used for computational estimation, but not nearly as much as reformulation (33) (see Table 5). Recall that translation is the process of changing the structure of the problem to enhance the mental management. For example, consider problem 7, about purchasing 8,483 hotdogs for $.60 each. Five students produced reasonable estimates using either reformulation or translation processes. The translation process adjusted the $.60 to $.50 and recognized that they could divide 8,400 hotdogs by two. These students translated a fairly complex multiplication by a decimal fraction into a much simpler division by two problems.

The least used estimation process was compensation (5), which is a process of adjusting for errors introduced by translation or reformulation (see Table 5). Consider problem 3, $8127\overline{)474257}$ and three students produced reasonable estimates. A student described a reasonable estimate as, "It will be 40 divided by 8 and I'll get 5 so the answer is 50 but I'll make it 55." This student used reformulation first by simplifying the numbers for mental manipulation and then compensated by added 5 (or 10%) to the estimated amount.

How did their estimation processes compare for straight computation and application problems?

If we compare the number of reasonable estimates for timed tests only, for straight computation (8) versus application (15) problems, the students' performance was almost doubled (see Table 3 and Table 4). However, that performance difference is not present for the un-timed test, comparing straight computation (23) and application (25) problems. Interestingly, examining estimation processes used to generate reasonable estimate for straight computation problems (1-5) versus application problems (6-10), the differences are negligible (see Table 5).

Rounding was a frequently observed student estimation strategy, referenced by students thinking aloud, and used across the three estimation processes for both problem types (straight computation and application). The students' truncation strategies appeared to be used to allow for more mental dexterity during computation. This approach appeared most useful for problems involving larger numbers, such as those in problems 3 (described in the previous section) and 8. The exact number of digits used depended on the size of the numbers and the operations; these two problems used the division operation. The example described above for Problem 3, the student used truncated numbers unlike the approach taken by the student for problem 8, which read: "The 1979 Super Bowl netted $21,319,908 to be equally divided among the 26 NFL teams. About how much does each team receive?" Four students produced acceptable estimates for the un-timed test by making the computation more mentally manageable; three students used

$\frac{21{,}000{,}000}{30}$ and one used $\frac{26{,}000{,}000}{26}$ then compensated their final estimates.

In summary, the findings from this study suggested that these students used reformulation estimation process more than either translation or compensation processes. Interestingly, when students were timed, they seemed to be able to perform at a higher level providing reasonable estimates when problems were situated in context (i.e., application problems). These findings may be due in part to this group of students' fragile computational fluency. The students in this study did not use different estimation processes based on the problem type. This was a very small study with ten students and these findings should not be generalized. However, the study design and analysis are sufficiently described allowing replication to discover the robustness of the findings on larger and different samples.

DISCUSSION

The purpose of this research was to identify computational estimation processes used by eighth-grade students and to determine differences in their estimation approaches to straight computation and application problems. They did not vary in the type of estimation processes used to estimate, and relied heavily on one process, reformulation with rounding and/or truncation strategies. These approaches enabled the students to exploit their strengths, adding and multiplying using smaller numbers. This suggests that students' computational estimation processes may be enhanced by improving their computational fluency. There was some evidence from students' computational estimation approaches supporting their understanding of place value and suggests some level proficiency with respect to number sense.

One implication of this study is the need to create opportunities for students to develop both number sense and computational fluency as is suggested by CCSSM (NGA, 2010). This study provides a specific rationale for middle school mathematics teachers to invest in explicitly teaching both computational fluency and number sense, but also suggests an opportunity for regular formative assessment through estimation as a way to monitor students' progress. According to Van de Walle (2006), students will not acquire the skills need to estimate without a deep understanding of numbers and their meaning.

The students demonstrated producing a greater number of reasonable estimates during the un-timed ACE test than during the timed test, but this was expected. However, what was not expected was that students produced a greater number of reasonable estimates for application problems than for straight computation problems during the timed test. The students' performance for application problems 7, 9, and 10 were the highest of the five application problems. The numbers in these problems tended to be relatively smaller than those in the other two application problems. While, this further evidences the need for computational fluency and number sense, perhaps this supports a second implication of this study, the need for teaching explicitly computational estimation processes such as, translation and compensation in ways that affords students greater flexibility with larger numbers (NCTM, 2000).

Although our sample size was small, we can assume that if students are equipped with greater computational fluency and facility with computational estimation processes and strategies perhaps the type of problem (i.e., straight computation and application) would not appear to be of significance when faced with time-constrained computational estimation opportunities.

This study was situated within an urban school setting, and there are implications that result with respect to mathematical opportunities for students. Haberman (2005) contends that many urban school environments focus on a "pedagogy of poverty" that highlights a methodology of giving and receiving information, assigning textbook work and seatwork and then moving to homework. Haberman's described pedagogy does not encourage creativity and innovation, skills that influence computational estimation processes and strategies. Nor does this limiting pedagogy promote reasoning or problem solving approaches that would afford students to successfully tackle real world application problems (National Research Council, 2011). Computational estimation is an essential element of mathematical literacy and is foundational for problem solving and reasoning (NGA, 2010). Because mathematics is always subject to making computational errors, good problem solvers make use of estimation for judging the reasonableness of solutions as a part of mathematical practice. If we are to improve mathematical learning and teaching in urban settings, focusing on computational estimation may be a good place to start.

All educators should be aware that students need to be provided with more flexible strategies when estimating. This may be a factor of the teacher's personal mathematics understanding. The NCTM (2007) states best, ". . . exploring what goes on in the mathematics classroom is essential to identifying issues and looking for opportunities for improvement" (p. 3). Also, students need to learn how to estimate within the context of mathematics and not as an isolated chapter or units. Students must be taught how to and afforded opportunities for applying estimation in their daily lives. This research raises several researchable questions. The following suggestions are offered as basis for framing related research:

- Investigate the relationship among estimation processes and strategies and the characteristics of problems (e.g., magnitude of numbers, operations, and complexity).
- Investigate the role that technology has on estimation strategies and reasonableness of estimates.
- Investigate the mathematics teaching pedagogies in urban classrooms and the opportunities and experiences afforded students.

Although many schools have moved toward a more stringent mathematics curriculum, flexibility, and innovative thinking must be addressed and practiced. Curriculum standard documents alone, will not impact the teaching and learning that occurs in the mathematics classroom (NCTM, 2007).

REFERENCES

Bestgen B, Reys R, Rybolt J, Wyatt W (1980). Effectiveness of systematic instruction on attitudes and computational estimation skills of preservice elementary teachers. *Journal for Research in Mathematics Education, 11*(2), 124-136.

Booth J, Siegler R (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 42*(1): 189-201.

Cochran J, Hartmann Dugger M (2013). Taking the guesswork out of computational estimation. *The Mathematics Educator, 23*(1): 60-73.

Dowker A, Flood H, Griffiths L, Harris L, Hook L (1996). Estimation strategies of four groups. *Mathematical Cognition, 2*(2), 113.

Dowker AD (1992). Computational estimation strategies of professional mathematicians. *Journal for Research in Mathematics Education, 23*(1), 45-55.

Edwards A (1984). Computational estimation for numeracy. *Educational Studies in Mathematics, 15*, 59-73.

Forrester MA, Pike CD (1998). Learning to estimate in the mathematics classroom: a conversation-analytic approach. *Journal for Research in Mathematics Education, 29*(3), 334-356.

Haberman M (2005). *Star teachers: The ideology and best practice of effective teachers of diverse children and youth in poverty*. Houston, TX: The Haberman Educational Foundation.

Hanson SA, Hogan TP (2000). Computational estimation skill of college students. *Journal for Research in Mathematics Education, 31*(4), 483-499.

Harper NW, Daane CJ (1998). Causes and reduction of math anxiety in preservice elementary teachers. *Action in Teacher Education, 19*(4), 29.

Heege H (1985). The acquisition of basic multiplication skills. *Educational Studies in Mathematics, 16*, 375-388.

Hogan TP, Brezinski KL (2003). Quantitative estimation: One, two, or three abilities? *Mathematical Thinking and Learning, 5*(4), 259-280.

Hope JA, Sherrill JM (1987). Characteristics of unskilled and skilled mental calculators. *Journal of Research in Mathematics Education, 18*(2), 98-111.

Jackson CD, Leffingwell RJ (1999). The role of instructors in creating math anxiety in students from kindergarten through college. *Mathematics Teacher, 92*(7), 583.

Lacey TA, Wright B (2009). Occupational employment projections to 2018. *Monthly Labor Review, 132*(11), 82-123. Retrieved from http://www.bls.gov.opub/mlr/2009/11/art5full.pdf

LeFevre J-A, Greenham SL, Waheed N (1993). The development of procedural and conceptual knowledge in computational estimation. *Cognition and Instruction, 11*(2), 95-132.

Levine DR (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education, 13*(5), 350-359.

Markovits Z, Sowder J (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25*(1): 4-29.

National Council of Teachers of Mathematics (2007). *Mathematics teaching today*. Reston, VA: Author.

National Council of Teachers of Mathematics (Ed.). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Governors Association. (2010). *Common core state standards.*

National Research Council. (2011). *Successful K-12 STEM education.* Washington, DC: The National Academies Press.

Patton MQ (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage.

Reys R, Bestgen B, Rybolt J, Wyatt J (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education, 13*(3), 183-201.

Reys R, Bestgen B, Rybolt J, Wyatt J (1980). *Identification and characterization of computational estimation processes used by in school pupils and out-of-school adults*. Washington, DC: National Institute of Education.

Reys BJ, Reys RE, Hope JA (1993). Mental computation: A snapshot of second, fifth, and seventh grade student performance. *School Science and Mathematics,* 93, 306-315.

Rounding. (n.d.). *Dictionary.com Unabridged*. Retrieved from Dictionary.com website: http://dictionary.reference.com/browse/rounding

Rubenstein RN (1985). Computational estimation and related mathematical skills. *Journal for Research in Mathematics Education, 16*(2)*,* 106-119.

Sowder JT (1989). Developing understanding of computational estimation. *Arithmetic Teacher, 36*, 25-27.

Sowder JT, Wheeler MM (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education, 20*(2), 130-146.

Truncate. (n.d.). *Dictionary.com Unabridged*. Retrieved from Dictionary.com website: http://dictionary.reference.com/browse/truncate

Tsui JM, Mazzocco MMM (2007). Effects of math anxiety and perfectionism on timed versus untimed math testing in mathematically gifted sixth graders. *Roeper Review, 29*(2), 132-139.

U.S. Department of Education. (2001*). Introduction: No child left behind.* Retrieved from http://www.ed.gov/nclb/overview/intro/index.html

Van de Walle J, Karp K, Bay-Williams J (2012). *Elementary and middle school mathematics: Teaching developmentally, student value edition* (8th ed.). Boston, MA: Pearson.

Van De Walle J (2006). *Elementary and middle school mathematics* (6th ed.)*.* Boston, MA: Pearson.

Van de Walle J, Folk S (2005). *Elementary and middle school mathematics* (Canadian ed.). Toronto, Canada: Pearson